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Dimensionless magnitudes in the marginal areas of aerodynamics.

by Theodore von Karman.

Translated from: Z. f. Flugwiss., 4: 3-5 (1956).

Summary: This paper contains a list of characteristic parameters occurring in specific domains of the science of aerodynamics. Special emphasis is laid on the dimensionless parameters of aerothermochemistry, since this domain becomes of increasing interest also to aeronautical engineers.

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Modern aerodynamics has, in the past decades, extended its domain in an astounding manner. Certain areas heretofore exclusively investigated by physicists and chemists, are now treated by the aerodynamicist. It would seem interesting, for this reason, to assemble the most important dimensionless parameters occurring in these areas.

Classic aerodynamics considered the air as a nearly incompressible fluid. Accordingly, Reynolds' number was the most important parameter of similarity. Written in the form $Re = v L \rho / \mu$ (v = velocity, L = a characteristic length of the arrangement, ρ = density, μ = viscosity coefficient), this parameter expresses the ratio of inert to viscous forces. The nomenclature dates back to 1904 and Arnold Sommerfeld (1). I pointed out in 1923 (2) that Reynolds' number may be interpreted as the product of two relations, $v/a = M$ (a = speed of sound or molecules) and $L/l = Sm$ (l = mean length of free path). The Mach number M , the designation of which originated with J. Ackeret, has meanwhile attained enormous popularity. The relation L/l should be named after M. Smoluchowski who studied the transitional area between continuous fluid motion and molecular flow.

As the aerodynamics of compressible fluids saw further development, especially supersonic aerodynamics, it was natural that Reynolds' number and the Mach number were utilized as the most important parameters of similarity. The marginal area of molecular flow gained in importance when studies were commenced of the motion of bodies in very thin media, e.g. in the air at very high altitudes. Zahm, in the United States, has called this area "superaerodynamics." As H.S. Tsien has shown, the parameter indicating the occurrence of molecular flow, e.g. at the body's walls, is not the relation L/l , but δ/l , where the length δ corresponds to the order of magnitude of the marginal layer's thickness. It is easily

shown that this relation is proportional to the magnitude $\sqrt{Re/M}$. The air acts like a continuous fluid as long as $\sqrt{Re/M} \gg 1$; molecular flow occurs when $\sqrt{Re/M} \approx 1$. The square of this characteristic magnitude, Re/M^2 , may be interpreted as the relation between the time L/v in which the fluid passes a body with length L , and the mean time interval $1/a$ between the collision of one molecule with others.

The phenomena of atomized fluids have attracted the attention of numerous aerodynamicists; the importance of these manifestations for combustion processes and other technical applications of fluid mechanics is evident. Two dimensionless magnitudes can be listed here: $\rho v^2 L/S$ and $\mu v/S$ (S = specific surface tension of the fluid to be atomized). These magnitudes express the relation between inert or viscous forces and surface tension. The first is called Weber's number, cf. (3). It seems to me that F. Scheubel was the first to utilize the two magnitudes in connection with the problem of atomization in his doctoral dissertation at Aachen. The relation of the two magnitudes is identical with Reynolds' number.

The study of thermal transition -- especially in the sense of Reynolds' analogy between friction and thermal transition -- suggests the introduction of the relation of transport coefficients, e.g. for the transport of movement and thermal energy. The magnitude $Pr = \mu c_p / \lambda$ (c_p = specific heat, λ = thermal conductivity coefficient) is generally known as Prandtl's number.

When the viscosity constant in Reynolds' number is replaced by the thermal conductivity coefficient, Peclet's number $Pe = v L c_p \rho / \lambda$ is obtained. Sometimes Nusselt's and Stanton's numbers are spoken of; these, however, are special forms of the thermal transition coefficient and do not belong to the group of dimensionless magnitudes proper.

Many problems involve a third transport coefficient, namely the coefficient of mass transport or diffusion. We shall limit ourselves to diffusion that occurs in a gaseous mixture due to the presence of a concentration gradient of the components. For example, let us take a mixture with two components, a so-called binary mixture. The binary diffusion coefficient D_{12} has the same dimension as the kinematic viscosity μ/ρ and the magnitude $\lambda/c_p \rho$. The relations $\mu/\rho D_{12}$ and $\lambda/\rho c_p D_{12}$ can therefore be introduced as characteristic dimensionless parameters. The first magnitude is often referred to as Schmidt's number. It seems, however, that the scholars do not agree on the nomenclature. For example, F. Schultz-Grunow designated $\rho c_p D_{12} / \lambda$ as Schmidt's number. The quotient of the diffusion coefficient and temperature conductivity is cited as Lewis' number after Bernard Lewis. Lewis and Semenow apparently recognized the significance of this magnitude for the flame theory in independent studies.

We shall now discuss a marginal area of aerodynamics that has experienced a mighty advance in the past two decades and which I should like to call aerothermochemistry. G. A. Crocco has suggested the term aerothermodynamics for the field combining aerodynamics and thermal chemistry. Aerothermochemistry is to elaborate on aerothermodynamics in the sense that chemical changes (reactions) occur in the flowing medium. Chemical changes may occur in an originally homogeneous gas flow. In the case of hypersonic flows (at very high Mach numbers), temperatures often occur that cause dissociation of the gas. When we observe such a flow, we obviously must consider dissociation and recombination of atoms. The science of combustion in flowing gas and the science of detonation also belong to aerothermochemistry.

Various authors who studied the laws of similarity pertaining to combustion, have suggested dimensionless combinations of characteristic magnitudes of combustive manifestations (4). First of all I want to mention G. Damkoehler (5), who in 1936 listed a number of similarity groups. S. Penner has recently held two of Damkoehler's parameters to be particularly useful in a study presented at the congress on model regulation in Venice.

One parameter represents the product of the time element L/v and the reaction frequency U . If we introduce the reciprocal value of the reaction frequency as reaction time $1/U$, LU/v equals the relation of the time element L/v to the reaction time. This magnitude really seems to be useful and clear for the understanding of practical problems. If, for example, L is the length of a combustion chamber, possibly the cylinder of a rocket with liquid propellant, and v equals the mean velocity at which the vaporized fuel passes through the chamber, then L/v is proportional to the so-called "residence time," i.e. the time spent by the fuel in the chamber. It is obvious that the completeness of combustion depends on this relation.

The second magnitude of Damkoehler which according to Penner deserves general attention, contains the heat tone q connected with the reaction. It is defined as the difference between the formation enthalpies of the components present in the mixture before and after the reaction, measured at a certain temperature. Damkoehler lists the dimensionless magnitude $qLU/vc_p T_0$ as an essential parameter of similarity. If we consider a combustion apparatus whose linear measurements are designated by the length L , the heat introduced through combustion during the time period will be proportional to $\rho_0 L^3 U q$, whereas the amount of heat introduced with the gas by convection during the time period will be proportional to $\rho_0 L^2 v c_p T_0$, where v is a characteristic velocity, c_p the specific heat, T_0 the temperature and ρ_0 the density of the introduced mixture.

This magnitude of Damkohler's may obviously be considered as a product of two factors, of which we have already discussed the first, $1/\lambda/c_p$. The second, $q/c_p T_0$, has the simple meaning that it represents the relation of heat tone (reaction enthalpy) to the thermal enthalpy, in which both magnitudes are applied to the mass of the original mixture.

The most important parameter that occurs in combustion or detonation phenomena, in my opinion, is the relation of the time scales that identify the purely mechanical-thermodynamic processes -- as viscosity, diffusion, heat conductivity -- on one hand, and the chemical processes on the other. The mean time interval between the collision of molecules in a gas or a gas mixture is proportional to the magnitude $1/a$ (1 = mean length of path, a = molecular velocity). We can also write this magnitude as μ/p or $\lambda/c_p p$. The time period characteristic for the chemical reaction is conversely proportional to the rate of reaction. The rate of reaction is generally defined as the number of moles converted in a given time and volume by chemists who are not afraid of complicated dimensional combinations. If we call the rate of reaction r , the "reaction time" becomes proportional to $\rho/M_m r$, where ρ is the density and M_m a mean molecular weight. The magnitude ρ/M_m is obviously the mean number of moles in the volume unit. We now introduce the relation

$$a = t_{\text{mech}}/t_{\text{chem}} = \lambda M_m r / \rho c_p p$$

($t_{\text{mech}} \approx \lambda/c_p$, $t_{\text{chem}} \approx \rho/M_m r$) as a characteristic dimensionless parameter. For slow, i.e. non-explosive reactions, $a \ll 1$. We shall now consider a combustive process with stationary propagation, accomplished best by observing the flow relative to the propagating wave. This unidimensional, stationary flow and the mechanical, thermal and chemical processes connected therewith are governed by the preservation of the material, the element of movement, and energy. The velocity of wave propagation equals the rate v_0 at which the unexpended mixture is supplied, and v_0/M_0 is designated as the Mach coefficient M_0 of the flow.

Now, if the condition $a \ll 1$ is observed in the basic equations of aerothermochemistry, i.e. in the continuity equations of the reaction partners, in the diffusion equations and in the equations for the maintenance of the elements of movement and energy, the following statements result:

1. If both $a \ll 1$ and $a/M_0^2 \ll 1$, first approximations may disregard those segments containing the coefficients of viscosity, diffusion and heat conductivity in favor of those determining changes in density, temperature, velocity and pressure according to the laws of the thermodynamics of ideal gases. Such a process, in which $a \ll 1$, $M_0 \gtrsim 1$, is called a detonation wave. The characteristic sign of the process is a shock wave preceding the combustion zone. The heating of the mixture permits combustion and, owing to the different orders of magnitude in the time intervals ($t_{\text{mech}} \ll t_{\text{chem}}$), combustion may be ignored during the shock.

2. If $a \ll 1$, but $a/M_0^2 \approx 1$, the forces of viscosity and dissipation corresponding to the viscosity, may still be disregarded. The equation for the maintenance of movement yields approximately the condition of constant pressure due to $M_0^2 \ll 1$. In equations of continuity, on the other hand, diffusion plays a decisive role, as does heat conductivity in the energy equation. Such a process is called a deflagration wave or a slow combustion wave.

The propagation rate of a slow combustion wave resulting from the above equations — without regard for a possible turbulence and the heightened diffusional and heat conductive processes connected therewith — is known as the laminar flame velocity. The theory of laminar flame velocity was initially developed in the form of a purely "thermal theory" without recourse to diffusion. In this case it is shown that the parameter a/M_0^2 offers a very useful service by yielding a dimensionless relation between the flame velocity and the characteristic values of the process. (The discussion of the quantities a and a/M_0^2 was first given in 1953 as part of my lectures at the Sorbonne and reproduced in my paper (6). The magnitude a/M_0^2 is essentially identical with one of Schultz-Grunow's similarity groups).

The determination of flame velocity is an eigenvalue problem. The quantity of mass flow in the time period must be determined by a standard cross section that permits the satisfaction of marginal conditions before and after the combustive process. The mass flow is a constant throughout the entire flame zone, so that

$$\rho_0 v_0 = \rho v = \text{const}$$

is valid for any arbitrary point.

Now the magnitude

$$\frac{a}{M_0^2} \approx \frac{\lambda \bar{m}_m r}{\rho c_p p} \frac{a^2}{v^2} \approx \frac{\lambda \bar{m}_m r}{c_p \rho^2 v^2}$$

represents a dimensionless combination of $(\rho v)^2$ and $\lambda \bar{m}_m r / c_p$. It follows that — ceteris paribus — the mass flow ρv becomes proportional to $\lambda \bar{m}_m r / c_p$.

The significance of this assertion is this: The eigenvalue of mass flow becomes the product of magnitude $\sqrt{\lambda \bar{m}_m r / c_p}$ (taken for a certain temperature, e.g. temperature T_f corresponding to the ultimate condition) and certain functions of the temperature and the concentration conditions. These functions depend on the changes in quantities λ , \bar{m}_m , r and c_p with the temperature and the concentrations.

In any case, the well-known rule for the pressure-dependability of the flame velocity may be derived therefrom. We consider a simple reaction in which each of the "partners" occurs with a molecular number $\nu_1, \nu_2, \nu_3, \dots$ (I have borrowed the amusing expression "reaction partner" from a paper by O. Lutz that appeared in this publication, 3 (1955), p. 151-159.) It is then generally assumed that the rate of reaction may be split up into a constant K , a temperature function $F(T)$ and a product $\prod c_i^{\nu_i}$.

c_i represents concentrations, i.e. moles per unit of volume. We therefore write

$$r = KF(T) \prod c_i^{\nu_i}$$

The sum $n = \sum \nu_i$ indicates the order of reaction.

If we introduce the dimensionless molar relations $\chi_i = c_i/c_r$, where $c_r = p/RT$ indicates the number of moles in the unit of volume, we get

$$\rho^2 v^2 = \rho_0^2 v_0^2 \approx \frac{\lambda \text{Lm}}{c_p} K F(T) \frac{p^n}{(RT)^n} \prod \chi_i^{\nu_i}$$

Since ρ_0 is proportional to the pressure, we get the result that — again ceteris paribus — the flame velocity is proportional to the $\lfloor (n/2)-1 \rfloor$ th power of the pressure.

It must be pointed out that, if diffusion is being considered, Lewis' number, i.e. the relation between the diffusion coefficient and the value of heat conductivity, exerts a decisive influence on flame velocity. Moreover, the simple laws indicated here are not valid without additional interpretation for simultaneous reactions, especially chain reactions.

Finally, I should like to mention a few additional areas that suggest the introduction of further dimensionless magnitudes. Aerodynamic problems dealing with the gravitational field profit by the introduction of Froude's number which is well-known in hydraulics and the theory of gravitational waves. Problems of natural convection utilize Grashof's number which actually represents a combination of Froude's and Reynolds' number, where the gravity acceleration is replaced by the relative gravitational difference of heated air. If Froude's number is written in the form $F = v^2/Lg$, Grashof's number is $Gr = Re^2 \beta \Theta / F$ (β = expansion coefficient of the fluid, Θ = temperature elevation).

A thoroughly modern branch of aerodynamics is the cosmic variety of our science, often called magnetohydrodynamics and concerned with the flow of thin, electrically charged media (7). In equations that, for example, describe so-called Alfvén's waves, the reciprocal value of electrical conductivity plays the role of viscosity inhibitor in the

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sense that the waves would propagate uninhibitedly in the case of infinite conductivity. Indeed, the magnitude c^2/σ (c = speed of light, σ = conductivity) has the same dimension as the kinematic viscosity μ/ρ . Hence a parameter may be introduced with a magnitude analogous to Reynolds' number: $Ba = \nu L \sigma / c^2$. This magnitude ought to enter the literature as Batchelor's number.

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